The important special case of a uniform normal force $\bar{S} = q_0 a\bar{n}$ is obtained by equating S_1^* and S_2^* to zero in Eq. (9).

In the case of two-dimensional problems, we may employ essentially the same procedure described above; i.e., a curved boundary in the deformed body is replaced by a collection of straight line segments, as indicated in Fig. 2. In this case, if $\bar{S}$ represents a uniform applied surface traction per unit length and $\bar{n}$ and $\bar{e}$ are unit vectors normal and tangent to the deformed surface,

$$\bar{S} = L^*(q_0\bar{n} + S\bar{e}) \tag{10}$$

where q_0 and S are normal and tangential tractions per unit length, and

$$L^* = |\bar{R}^2 - \bar{R}^1| = [(x_\alpha{}^2 + u_\alpha{}^2 - x_\alpha{}^1 - u_\alpha{}^1) \times (x_\alpha{}^2 + u_\alpha{}^2 - x_\alpha{}^1 - u_\alpha{}^1)]^{1/2} \tag{11}$$

$\alpha = 1, 2$. It is easily verified that

$$\bar{n} = 1/L^*(L_2^*\bar{i}_1 + L_1^*\bar{i}_2) \tag{12}$$

$$\bar{e} = -1/L^*(L_1^*\bar{i}_1 - L_2^*\bar{i}_2) \tag{13}$$

where $L_i^* = L_1 + u_1{}^2 - u_1{}^1$ and $L_2^* = L_2 + u_2{}^2 - u_2{}^1$. Here $L_1 = x_1{}^2 - x_1{}^1$, $L_2 = x_2{}^2 - x_2{}^1$ are the projections of the line drawn from node 1–2 on the x_1 and x_2 axes in the undeformed element. Then the associated generalized forces are simply

$$\begin{aligned} \hat{p}_{N1} &= \tfrac{1}{2}[q_0(L_2 + u_2{}^2 - u_2{}^1) - S(L_1 + u_1{}^2 - u_1{}^1)] \\ \hat{p}_{N2} &= \tfrac{1}{2}[q_0(L_1 + u_1{}^2 - u_1{}^1) + S(L_2 + u_2{}^2 - u_2{}^1)] \end{aligned} \tag{14}$$

for all N, $N = 1, 2$. In the special case of purely normal loading ($S = 0$), the forces $\hat{p}_{N\alpha}$ of Eq. (14) reduce to $(-1)^{\alpha}\epsilon_{\alpha\beta}q_0L_\beta^*/2$, as given in Ref. 3, wherein α, $\beta = 1, 2$, and $\epsilon_{\alpha\beta}$ being the two-dimensional permutation symbol.

Sample Calculations

We cite briefly the results of applying the approximate equations for generalized forces to a representative problem involving finite deformations. The problem considered is the inflation of an initially flat, thick, simply supported, rubber circular plate, 15 in. in diameter and 0.5 in. thick, subjected to a steadily increasing uniform internal pressure. The loading surface of the plate was idealized using 10 finite elements over which the displacement fields are assumed to be quadratic. Solutions obtained using the exact equations for generalized forces were compared with approximate forces computed by idealizing the loading surface with 20 flat triangular elements. The maximum deviation was less than one %. To emphasize the importance of accounting for the effects of deformation of the boundary elements in computing generalized forces in finitely deformed structures, the variation the orientation of a typical boundary element and its cross-sectional area with internal pressure is indicated in Fig. 3. Clearly, the effects of finite distortion of the loading surface may be equally as important as the rotation of the element.

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Film Cooling Effectiveness in Hypersonic Turbulent Flow

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A PAUCITY of experimental information prevents an accurate evaluation of film cooling as an active system for hypersonic flight vehicles.[1] Available experimental data (mainly for subsonic mainstream flow) when extended to hypersonic speeds indicated that film cooling will not be competitive with other active cooling systems[1]; however, recent investigations[2,3] at Mach 6 show significantly greater effectiveness for film cooling in high-speed turbulent flow than in low-speed flow (see Ref. 2 for a summary of low-speed results). The present Note reports accurate measurements of surface equilibrium temperature downstream of a rearward-facing, two-dimensional slot with tangential mass (air) injection into a thick, hypersonic turbulent-boundary layer.

An insulated flat-plate model with end plates, shown in Fig. 1, was mounted parallel with the tunnel wall of the Langley 20-in. Mach 6 wind tunnel and alined with the bottom lip of the slot. Surface temperatures were measured by flush mounted thermocouples located along the centerline of the insulated plate surface. The slot configuration, shown in the insert of Fig. 1, allowed for slot heights (s) of 0.159, 0.476, and 1.114 cm with a constant lip thickness (t) of 0.159 cm. The slot mass flow rate ($\rho_j u_j$) was uniform over a midspan of at least 21 cm and regulated so that the ratio of measured slot mass flow rate to calculated freestream mass flow rate ($\lambda = \rho_j u_j/\rho_\infty u_\infty$) ranged from 0.06 to 1.60; the flow at the throat of the slot was always sonic and the ratio of slot velocity to freestream velocity (u_j/u_∞) was approximately 0.34. The freestream total temperature ($T_{t,\infty}$) and unit Reynolds number per cm (R/cm) were 472°K and 0.241×10^6, respectively. The total temperature of the slot flow ($T_{t,j}$) varied slightly from run to run (278° to 294°K). From previous measurements the tunnel-wall boundary layer over the slot was known turbulent and approximately 5.08-cm thick with a momentum thickness Reynolds number approximately 3.8×10^4. The tunnel-wall temperature to total temperature ratio was approximately 0.70.

Experimental Results

A summary of the equilibrium temperatures measured downstream of the slot is presented in Fig. 2 in a form which correlated similar data in Ref. 2. The equilibrium temperatures are expressed as a film cooling effectiveness parameter ϵ, where

$$\epsilon = T_{t,\infty} - T_{\mathrm{eq}}/T_{t,\infty} - T_{t,j}$$

and T_{eq} the equilibrium surface temperature. The effectiveness is correlated with a distance parameter $(x/s)\lambda^{-0.8}$, where x is chordwise distance downstream of the slot. For these data, local surface temperature was considered to be in equilibrium when for a period of at least 100 sec. This temperature changes less than 0.1%. Only the forward portion of the plate surface had in general reached equilibrium temperature in the test times available ($\approx$1000 sec); thus, equilibrium temperatures at large x/s were not obtained. Data are presented for three slot heights and injection flow rates ranging from $\lambda = 0.06$ to $\lambda = 1.6$. The effectiveness parameter correlates in a relatively narrow band with $(x/s)\lambda^{-0.8}$ for the slot mass flow rates and slot heights of this investigation. The best straight line fairing of the data in Fig. 2 for $(x/s)\lambda^{-0.8}$

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> 80 has the equation

$$\epsilon = 2.66\ [(x/s)\lambda^{-0.8}]^{-0.22}$$

Thus, the cooling length ($\epsilon = 1$) is expressed as

$$x_{CL}/s = 85\lambda^{0.8}$$

where the cooling length (x_{CL}/s) is the maximum value of x/s for which the surface temperature downstream of the slot is equal to or less than the total temperature of the slot flow ($T_{t,j}$). Included in the figure is the prediction from an implicit finite-difference method[4]; this method was applied by Bushnell and Beckwith to slot flow in a manner similar to that in Ref. 5 except that the mixing-length distribution was modified to account for the slot flow and mixing-region downstream of the slot. Predictions from Ref. 5 apply only far downstream of the slot, whereas the present prediction applies close to the slot location as well. The present prediction is for $s = 0.159$ cm and the static pressure at the slot exit equal to the freestream static pressure ("matched" pressure condition). Experimental data for the matched pressure condition correlate well with data for higher slot exit pressure to freestream static pressure ratios when compared as in Fig. 2, and the finite-difference prediction agrees with the level and trend of the present data.

The only other investigations of hypersonic film cooling effectiveness with tangential injection were also at Mach 6. A fairing of the film cooling effectiveness data obtained from heat-transfer measurements in Ref. 2 generally falls below the present results as shown in Fig. 2. The disagreement between the two sets of data is unexpected since the flow conditions were similar. Part of the difference in the correlations may be caused by the data reduction technique used in Ref. 2 (equilibrium temperatures were inferred from surface heat-transfer measurements) and part to the difference in experimental configurations (a slight injection angle for Ref. 2). A fairing of effectiveness obtained from unpublished direct measurements of surface equilibrium temperature (supplied by V. Zakkay of New York University) for the same conditions and configuration as Ref. 2 is also shown in Fig. 2. Although the effectiveness obtained from direct temperature measurements using the configuration of Ref. 2 better agrees with the present results than the fairing from Ref. 2, a significant disagreement still exists for $(x/s)\lambda^{-0.8} > 200$. Included in Fig. 2 for comparison is a fairing of data from Ref. 3 for tangential supersonic slot injection (slot Mach number at the slot exit equal to 2.3) into a Mach 6 stream. The slope of the present data substantially agrees with that from Ref. 3 for $(x/s)\lambda^{-0.8} > 300$. Although significant differences exist between the values of film cooling effectiveness found herein and from other hypersonic investigations, all the results show more efficiency for film cooling in hypersonic turbulent flow than would be inferred by extrapolation of subsonic results.

In summary, this investigation of the effect of sonic slot injection into a thick turbulent-boundary layer on the downstream equilibrium temperature confirms that film cooling in hypersonic flow is significantly more efficient than previous extrapolations of lower-speed results had indicated. In addition, the present results indicate a greater film cooling effectiveness than found in other hypersonic investigations. Thus, a new evaluation of film cooling as an active cooling system for use on hypersonic vehicles is desirable.

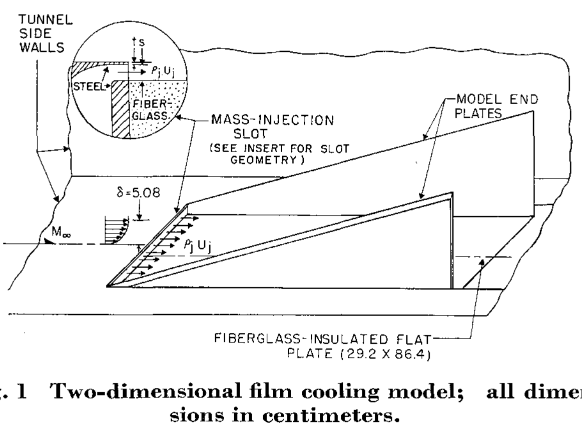

Fig. 1 Two-dimensional film cooling model; all dimensions in centimeters.

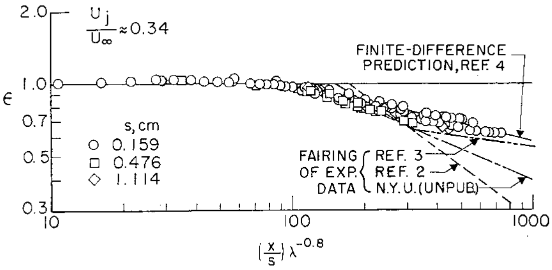

Fig. 2 Film cooling effectiveness at Mach 6.

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An Improved Conjugate Direction Minimization Procedure

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THE more advanced parameter optimization processes, e.g., Davidon-Fletcher-Powell,[1] are based upon generation of conjugate direction sequences and/or inference of the second partial derivative matrix inverse. A method having both of these properties and, in addition, being free of one-dimensional minimization requirements would be advantageous. In attempting to synthesize such a method, the writers have come to recognize that a "batch processor" version of DFP considered in an earlier study[2] does, in fact, combine the desirable properties by mere deletion of the one-dimensional search. This version will be reviewed, and an argument advanced for conjugacy of the directions generated in the ab-

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